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AUTOMODEL FLOWS OF RELATIVISTIC GAS IN CASE OF
POINT SYMMETRY IN THE GENERAL THEORY OF
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SUMMARY

All possible automodel flows of relativistic gas in the general theory of relativity, when endowed with point symmetry, offer interest for various problems of astrophysics and cosmology. In the present paper formulas are derived for the case of general automodel flow, for the ultra-relativistic gas flow and for the case of dust-like matter in its own gravitational field.

* * *

All the parameters of a flowing gas for a given equation of state, together with the metric, having in this case the form [1]

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1)$$

where ν and λ are functions of r and t , are described by the Einstein equations $R_i^k - 1/2 \delta_i^k R = \kappa T_i^k$ and by the equations of motion stemming from them:

$$T_{i;k}^k = 0. \quad (2)$$

The general investigation of the automodel state of the indicated system of equations is conducted by a method analogous to the finding of automodel solutions for gas flow in the special theory of relativity (STR) [2, 3].

From the system (2), we have for the adiabatic flow of gas

$$\frac{d(wu_i)}{ds} + \frac{\partial w}{\partial x^i} = \frac{w}{2} u^k u^l \frac{\partial g_{kl}}{\partial x^i} + T \frac{\partial \varepsilon}{\partial x^i}, \quad (3)$$

$$\frac{\partial}{\partial x^k} \left(\frac{u^k}{V} \sqrt{-g} \right) = 0, \quad \frac{d\varepsilon}{ds} = 0.$$

3a. * AVTOMODEL'NOYE DYIZHENIYA RELATIVIVISTKOGO GAZA V OBNHCHEY TEORII OTNOSHIT'L'NOSTI V SLUCHAYE TOCHECHNOY SIMMETRII

Here u^i is the 4-velocity; w and σ are respectively the heat content and the entropy that may be expressed through the pressure P and the specific volume V

$$w = \frac{k}{k-1} PV + \alpha c^2, \quad PV^k = \sigma. \quad (4)$$

It is appropriate to write further the system (3) in a form, resembling the system of equations for the adiabatic flow of gas in the STR*:

$$\begin{aligned} \left(\frac{\partial \ln w}{\partial r} + \frac{a}{c^2} \frac{\partial \ln w}{\partial \tau_0} \right) + \frac{1}{\theta^2 c^2} \left(\frac{\partial a}{\partial \tau_0} + a \frac{\partial a}{\partial r} \right) = - \frac{1}{2} \left(\frac{\partial v}{\partial r} + \frac{a}{c^2} \frac{\partial \lambda}{\partial \tau_0} \right) + \frac{T \theta^2}{w} \frac{\partial \sigma}{\partial r} - \\ - \left(\frac{\partial \ln V}{\partial \tau_0} + a \frac{\partial \ln V}{\partial r} \right) + \frac{1}{\theta^2} \left(\frac{\partial a}{\partial r} + \frac{a}{c^2} \frac{\partial a}{\partial \tau_0} \right) + \frac{2a}{r} = - \frac{1}{2} \left(\frac{\partial \lambda}{\partial \tau_0} + a \frac{\partial v}{\partial r} \right), \quad (5) \\ \frac{\partial \sigma}{\partial \tau_0} + a \frac{\partial \sigma}{\partial r} = 0. \end{aligned}$$

In the system (5), a is the RMS velocity, measured by proper time, $d\tau = \sqrt{-g_{00}} dt$, $\theta^2 = 1 - a^2/c^2$; further, an auxiliary time $d\tau_0 = e^{(\lambda-v)/2} dt$ is introduced with the view of curtailing the writing. At the same time, one should remember that the quantities τ_0 and r are not independent.

The system (5) differs from the equation of motion for a gas in STR by the presence of two functions λ and v describing the gravitational forces acting upon the gas in its proper gravitational field. For their determination one should make use of the Einstein gravitation equations which, for a centrally-symmetrical gas flow with metric (1), have the form brought out in [1]. From the indicated system of equations we may obtain the correlations

$$\begin{aligned} \frac{\partial \lambda}{\partial \tau_0} \left(1 + \frac{a^2}{c^2} \right) + a \left(\frac{\partial v}{\partial r} + \frac{\partial \lambda}{\partial r} \right) = 0, \quad \frac{\partial \lambda}{\partial \tau_0} = - \frac{\kappa (P + \varepsilon) r c^2 a}{\theta^2}, \\ \frac{\partial \lambda}{\partial \tau_0} + a \frac{\partial \lambda}{\partial r} = - a \left(\frac{e^\lambda - 1}{r} + \kappa P r e^\lambda \right). \quad (6) \end{aligned}$$

Eliminating v from (5) with the aid of (6) and expressing σ through P and V , we shall obtain

$$\begin{aligned} \frac{w}{\theta^2 c^2} (a_{\tau_0} + a a_r) + V \left(P_r + \frac{a}{c^2} P_{\tau_0} \right) = \frac{w}{2a} (a \lambda_r + \lambda_{\tau_0}), \\ - (V_{\tau_0} + a V_r) + \frac{V}{\theta^2} \left(a_2 + \frac{a}{c^2} a_{\tau_0} \right) + \frac{2aV}{r} = \frac{aV}{2} \left(\frac{a}{c^2} \lambda_{\tau_0} + \lambda_2 \right), \quad (7) \end{aligned}$$

$$\begin{aligned} w_{\tau_0} + a w_r = V (P_{\tau_0} + a P_r), \quad \lambda_{\tau_0} = - \frac{\kappa w r e^\lambda a}{V \theta^2}, \\ v_r = - \frac{1}{a} \lambda_{\tau_0} \left(1 + \frac{a^2}{c^2} \right) - \lambda_r, \quad w = \frac{k}{k-1} PV + \alpha c^2. \end{aligned}$$

[* STR means "special theory of relativity, vs GTR - general theory of relativity.]

According to the general rules for finding automodel solutions [1, we shall represent the functions searched for in the form

$$\begin{aligned} a &= \xi_1(z), \quad 1/V = t^{m_2} \xi_2(z), \quad P = t^{m_3} \xi_3(z), \\ e^\lambda &= t^{m_4} \xi_4(z), \quad e^\nu = t^{m_5} \xi_5(z), \quad z = r/t. \end{aligned} \quad (8)$$

We should pay attention to the fact that the automodel solution of (8) is constructed with the utilization of independent variables r and t — time of the central observer. Note that the system (8) is less general than the analogous equations in relativistic gas dynamics.

The presence in the equations (7) of multipliers θ^2 and w compels us to select the power 0 at t in the expression for a ; in the expressions for $1/V$ and P the powers of t must be chosen coinciding.

Substituting (8) into (7) we may arrive at a system of 5 ordinary equations for the unknowns $\xi_1 - \xi_5$. In the process of obtaining the indicated system of arbitrary powers at t are left ($m_2 = -2$, $m_3 = m_4 = 0$). The general automodel solution containing 5 unknown quantities (a, V, P, λ, ν), will thus contain 5 arbitrary constants entering into the general solution of the system of 5 ordinary equations.

Note that for the determination of gas parameters the first 4 equations would be sufficient (7). In this case the solution would include 4 arbitrary constants, that is, as many as are included in the automodel solution of gas flow in the STR [3]. The coincidence of the number of arbitrary constants is the consequence of the fact that the centrally-symmetrical motion of matter determines unambiguously the space metric.

It is easy to see that in the general case of automodel solution of (8), no automodel isentropic flow exists. Indeed, $\epsilon = PV^2 = \text{const.}$ superimposes on powers of m_2 and m_3 an additional condition, which contradicts the values of the same quantities found above.

Let us consider further the ultrarelativistic gas flow. The functions searched for will now be represented in the form

$$\begin{aligned} a &= \xi_1(z), \quad 1/V = t^{m_2} \xi_2(z), \quad P = t^{m_3} \xi_3(z), \\ e^\lambda &= t^{m_4} \xi_4(z), \quad e^\nu = t^{m_5} \xi_5(z), \quad z = r/t. \end{aligned} \quad (9)$$

When obtaining the system of ordinary equations, the following conditions will be superimposed on the powers

$$m_4 = m_5 = 0, \quad m_3 = -2. \quad (10)$$

Therefore, the power of m_2 at V remains arbitrary, which allows the fulfillment of isotropic condition by postulating $m_2 = -m_3/k$. The latter implies that in case of ultrarelativistic gas there exists for the system (7) an automodel isentropic flow; this is contrary to the general relativistic case where no such flow exists.

The systems of ordinary equations, obtained from (7) with the aid of (8) and (9) are not practical for research, as being too cumbersome. It is thus appropriate to effect the transformations of the system (7) and then make use of the general investigation if the above-indicated automodel solutions.

Let us pass in (7) to the new independent variables λ, r . At the same time, from (6) we have

$$e^{(v-\lambda)/2} a \frac{\partial t}{\partial \lambda} = -\frac{e^{-\lambda} r \theta^2}{\kappa r^2 (P + \varepsilon)}. \quad (11)$$

With the aid of the latter, we shall obtain from (7) the system

$$\begin{aligned} & \frac{1}{2\theta^2 c^2} \left[\frac{\partial a^2}{\partial \lambda} (e^{-\lambda} - 1 - \kappa r^2 P) + r e^{-\lambda} \frac{\partial a^2}{\partial r} \right] - \frac{\omega^2}{c^2} \left[r \frac{\partial \ln V}{\partial r} e^{-\lambda} + \right. \\ & \left. + \frac{\partial \ln V}{\partial \lambda} (e^{-\lambda} - 1 + \kappa r^2 \varepsilon) \right] = \frac{1}{2} (e^{-\lambda} - 1 - \kappa r^2 P) + \frac{T}{w} \frac{\partial \varepsilon}{\partial \lambda} \kappa r^2 (P + \varepsilon), \\ & - \left[\frac{\partial \ln V}{\partial r} r e^{-\lambda} + \frac{\partial \ln V}{\partial \lambda} (e^{-\lambda} - 1 - \kappa r^2 P) \right] + \frac{1}{2\theta^2} \left[r \frac{\partial \ln a^2}{\partial r} e^{-\lambda} + \right. \\ & \left. + \frac{\partial \ln a^2}{\partial \lambda} (e^{-\lambda} - 1 + \kappa r^2 \varepsilon) \right] + 2e^{-\lambda} = \frac{1}{2} [e^{-\lambda} - 1 + \kappa r^2 \varepsilon], \quad (12) \\ & \frac{\partial \varepsilon}{\partial \lambda} (e^{-\lambda} - 1 - \kappa r^2 P) + \frac{\partial \varepsilon}{\partial r} r e^{-\lambda} = 0, \\ & \frac{\partial v}{\partial r} r e^{-\lambda} + \frac{\partial v}{\partial \lambda} \left[\frac{\kappa r^2}{\theta^2} (P + \varepsilon) + e^{-\lambda} - 1 - \kappa r^2 P \right] = \\ & = \frac{a^2}{c^2} \kappa r^2 \frac{P + \varepsilon}{\theta^2} - (e^{-\lambda} - 1 - \kappa r^2 P). \end{aligned}$$

Let us consider further the isentropic flow of an ultrarelativistic gas, when

$$P = (k-1)\varepsilon, \quad P = \sigma V^{-k}, \quad \sigma = \text{const}, \quad \varepsilon(k-1) = \sigma V^{-k}. \quad (13)$$

As was noted above, λ, a, v are functions of \underline{z} ; hence, in order to obtain the automodel solution of (12), we should postulate

$$a = a(\lambda), \quad v = v(\lambda), \quad P = r^{-2} A_1(\lambda). \quad (14)$$

Substituting (13) and (14) into (12), we shall obtain

$$\begin{aligned} & \frac{1}{2\theta^2 c^2} \frac{da^2}{d\lambda} (e^{-\lambda} - 1 - \kappa A_1) - \\ & - \frac{\omega^2}{c^2} \left[\frac{2}{k} e^{-\lambda} - \frac{A_1^{-1}}{k} \frac{dA_1}{d\lambda} \left(e^{-\lambda} - 1 + \frac{\kappa A_1}{k-1} \right) \right] = \frac{1}{2} (e^{-\lambda} - 1 - \kappa A_1), \\ & - \left[\frac{2}{k} e^{-\lambda} - \frac{A_1^{-1}}{k} \frac{dA_1}{d\lambda} (e^{-\lambda} - 1 - \kappa A_1) \right] + \\ & + \frac{1}{2\theta^2} \frac{d \ln a^2}{d\lambda} \left(e^{-\lambda} - 1 + \frac{\kappa A_1}{k-1} \right) + 2e^{-\lambda} = \frac{1}{2} \left(e^{-\lambda} - 1 + \frac{\kappa A_1}{k-1} \right), \quad (15) \\ & \frac{dv}{d\lambda} \left(\frac{\kappa}{\theta^2} A_1 \frac{k}{k-1} + e^{-\lambda} - 1 - \kappa A_1 \right) = \frac{a^2}{\theta^2 c^2} \kappa \frac{k A_1}{k-1} (e^{-\lambda} - 1 - \kappa A_1). \end{aligned}$$

To conclude, let us consider the automodel motion of dust-like matter in its proper gravitational field. A problem of that sort has in its proper calculation system an exact solution [1]; however, it is interesting to obtain the same solution in the reading system of the central observer, who, besides matter density distribution observes also the distribution of velocities.

Postulating in (12) $P = 0$ and taking into account that $a = a(\lambda)$, we shall obtain from the first equation of the indicated system

$$a^2 / c^2 = 1 - e^{-\lambda} \quad (16)$$

(the integration constants was so chosen that at infinity at $\lambda \rightarrow 0$, $a \rightarrow 0$).

When we also take into account (16) and the correlations following from (8),

$$1/V = t^2 B(z) = r^2 F(\lambda);$$

we shall have from the second equation of the system (12)

$$\rho = 1/V = 1/r^2 [c_1 - \kappa c^2 F(z)], \quad (17)$$

$$z = \frac{e^{-\lambda}}{2}, \quad F(z) = \int \frac{e^{-z} dz}{(1-2z)}. \quad (18)$$

It follows from (18) and (17) that at $r \rightarrow \infty$ ($e^{-\lambda} \rightarrow 1$) $\rho \rightarrow 0$. Utilizing (16) and (17), we determine from the last equation of the system (12) $v(\lambda)$ after which from (11) we find $t(\lambda, r)$.

Note in conclusion that certain questions of automodel flows of gas in the GTR have been examined in the V. A. Skripkin dissertation [6] in connection with the investigation of shock waves.

**** THE END ****

REFERENCES

- [1].- L. D. LANDAU, E. M. LIFSHITS.- Teoriya polya (Field Theory)., M., 1962
 [2].- V. A. SKRIPKIN.- Dok. AN SSSR, 127, No. 2, 1959.
 [3].- K. P. STANYUKOVICH.- Ibid. 140, No. 1, 1961.
 [4].- L. I. SEDOV.- Metody podobiya is razemrnosti v mekhanike. (Similarity and Dimensionality Methods in Mechanics). M., 1957.
 [5].- K. P. STANYUKOVICH.- Neustanovivhseyesya dvizheniye sploshnoy sredy. (Unsteady motion of continuum), 1955.
 [6].- V. A. SKRIPKIN.- Dissertation, Moscow State Univ., 1962.

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